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by weight, of 1 of manganese, 100 of sulphur, 1000 of sand ; and on exposing ten grains of the mixed substances to a red heat on a slip of platina, the mixture afforded, with pure water, a solution of sulphate of manganese, which was rendered turbid by prussiate of potash. The method is applicable to organic substances, both vegetable and animal ; but the Professor's experiments on such bodies are still in progress.

The Rev. Charles Graves communicated a general theorem in the Calculus of Quaternions :

Let Q be a variable quaternion, of which $f(Q)$ is a homogeneous function of the n^{th} degree ; and let

$$S.df(Q) = S.NdQ,$$

then we shall have

$$S.NQ = nSf(Q). \quad (1)$$

And, more generally, if $Q, Q', Q'', \&c.$, be any number of variable quaternions, of which $f(Q, Q', Q'', \dots)$ is a homogeneous function of the n^{th} degree ; and if

$$S.df(Q, Q', Q'', \dots) = S.NdQ + S.N'dQ' + S.N''dQ'' + \&c.,$$

we shall have

$$S.NQ + S.N'Q' + S.N''Q'' + \dots = nS.f(Q, Q', Q'', \dots) \quad (2)$$

Let us first establish the theorem in a particular case, and it will afterwards be easy to show that the proof admits of extension to the most general one. Suppose, therefore, that

$$f(Q) = RQR'QR''$$

where R, R', R'' are any constant quaternions ; then

$$S.df(Q) = S.RdQR'QR'' + S.RQR'dQR'' ;$$

or, in virtue of the rule which permits us to execute a cyclic permutation on factors under the *scalar* sign,

$$\begin{aligned} S.df(Q) &= S.(R'QR'R + R''RQR')dQ \\ &= S.NdQ. \end{aligned}$$

Retracing the steps of this process, we see that

$$\begin{aligned} S.NQ &= S.(R'QR'R + R''RQR')Q \\ &= S.RQR'QR'' + S.RQR'QR'' \\ &= 2S.f(Q). \end{aligned}$$

And the proof would hold equally good if $f(Q)$ became the sum of any number of terms, all of the same form as $RQR'QR''$. The theorem is therefore proved for the most general homogeneous function of the second degree of Q . The nature of the proof remaining quite unaltered when we suppose n to become any other positive integer; and, moreover, conceive the function f to depend upon any number of variables, it seems unnecessary to occupy space with the fuller statement of it.

The equation (1) is an extension to the calculus of quaternions of the ordinary algebraic equation,

$$x \frac{d}{dx} x^n = nx^n;$$

and equation (2) is an extension of the more general theorem discovered by Fontaine, viz., that if U be a homogeneous function of the n^{th} degree of any number of variables, x, y, z , &c.,

$$x \frac{dU}{dx} + y \frac{dU}{dy} + z \frac{dU}{dz} + \dots = nU.$$

By means of this latter theorem it is proved that if a surface be represented by an equation $U = \text{const.}$, in which U is homogeneous, and of the n^{th} degree, in x, y , and z , the equation of its tangent plane at the point xyz will be

$$\frac{dU}{dx} x' + \frac{dU}{dy} y' + \frac{dU}{dz} z' = n \text{const.}$$

It was from observing the existence of a similar relation

between the equations of a surface of the second order and of its tangent plane, as found by Sir William R. Hamilton in his geometrical applications of the Calculus of Quaternions, that Mr. Graves was led to investigate the theorem now communicated to the Academy.

The following theorem respecting ellipsoids, obtained by the method of quaternions, was communicated by Sir William Rowan Hamilton, in a note to the Secretary of Council :

“ On the mean axis of a given ellipsoid, as the major axis, describe an ellipsoid of revolution, of which the equatorial circle shall be touched by those tangents to the principal section of the given ellipsoid (in the plane of the focal hyperbola), which are parallel to the umbilicar diameters. In this equatorial circle, and in every smaller and parallel circle of the new ellipsoid thus constructed, conceive that indefinitely many quadrilaterals are inscribed, for each of which one pair of opposite sides shall be parallel to the given umbilicar diameters, while the other pair of opposite sides shall be parallel to the asymptotes of the focal hyperbola. Then *the intersection of the first pair of opposite sides of the inscribed quadrilateral will be a point on the surface of the given ellipsoid.*

“ I may remark that the distance of either focus of the new ellipsoid from the common centre of the new and old ellipsoids, will be equal to the perpendicular let fall from either of the two points, which were called t and v in a recent note and diagram, on the umbilicar semidiameter au , or on that semidiameter prolonged; while the distance of the umbilic u from the foot of either of these two perpendiculars, that is, the projection of either of the two equal tangents to the focal hyperbola, tu , uv , on the umbilicar semidiameter au , or on that semidiameter prolonged, will be the minor semiaxis, or the radius of the equator, of the new ellipsoid (of revolution).